This case corresponds to $\tau^* = 0.175$ and an impulse of 21. The actual response history of the beam is shown in Fig. 4b.

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An Estimate of Mean Flow Properties in a Turbulent Diffusion Flame

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Introduction

SOME recent models for the computation of turbulent diffusion flames have included effects of fluctuations of concentration and temperature on the flow behavior.[1-6] These theories have neglected some second-order correlations involving density fluctuations, however, so that the level of modeling may not be completely consistent. For shear flows composed of species of widely differing molecular weights, density fluctuations may be large,[7] and some correlations involving them may not be negligible.[8]

This study considered a simpler approach, in which turbulent diffusion flames were computed while neglecting fluctuations of density, concentration, or temperature. An estimate for the time-average rate of consumption of the fuel specie $\bar{w}_f(\sec^{-1})$ was made, based mainly on dimensional analysis. First, it was assumed that mixing of the reactants is the rate-controlling step. This process occurs on the macroscale of the turbulence.[7] For a turbulent velocity scale, $k^{0.5}$, and a turbulent length scale, $k^{1.5}/\epsilon$, $\bar{w}_f$ can be assumed to be proportional to ϵ/k. Here k is the turbulent kinetic energy per unit mass, and ϵ is the rate of dissipation of turbulent energy per unit mass. This is essentially the form proposed by Spalding[9] for premixed turbulent flames. It also was assumed that $\bar{w}_f$ is a function of $\bar{C}_f$ and $\bar{C}_o$, the time-average mass fractions of fuel and oxidizer, respectively. In order that $\bar{w}_f$ goes to zero when either reactant disappears, a general form $\bar{w}_f = -A(\epsilon/k)\,\bar{C}_f^m \bar{C}_o^n$ was assumed, where A, m, and n are positive constants. Having three constants would seem to provide wide latitude in curve fitting a given set of experimental data. However, the results of computations suggest that the simple choice $m = n = 1$, with A determined empirically, may be adequate to represent the data.

Results

Calculations were made for the experimental conditions of Kent and Bilger.[10] The experimental arrangement was a central hydrogen jet issuing into a parallel, coaxial airstream. The initial and boundary conditions for the calculations were the same as those measured. Initial values for k were determined from the measured $\overline{u'^2}$ by $k \cong \overline{u'^2}/4\,C_\mu^{0.5}$, where $C_\mu \cong 0.09$ (Ref. 11). Initial values of ϵ were determined from $\epsilon \cong C_\mu^{0.5} k\,\partial\bar{U}/2r$.

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The flowfield solution used the time-averaged conservation equations for mass, momentum, energy, species mass fraction, k, and ϵ for axisymmetric flow in the boundary-layer approximation. Various correlations were modeled as described by Launder and Spalding.[12] Values of the empirical constants in the k and ϵ equations were also the same as those given in Ref. 12. A mixture of perfect gases with variable specific heats was assumed.

Results for two forms of the equation for $\bar{w}_f$ are given in what follows. The first and simplest form is

$$\bar{w}_f = -A(\epsilon/k)\,\bar{C}_f \bar{C}_o \tag{1}$$

Using the experimental data of Kent and Bilger[10] for the case where $\bar{U}_j/\bar{U}_e = 10$, the value $A = 12$ was found to give the best agreement with the measured centerline variations of temperature and of the mole fractions of H_2, H_2O, and O_2. These results are shown in Fig. 1. Here, $\bar{U}_j$ is the jet mass-average velocity and $\bar{U}_e$ is the mean freestream velocity. These results seemed to be relatively insensitive to the value of A, as variations of up to about 30% had only small effects on the computed flow properties.

Calculations also were made for the following form:

$$\bar{w}_f = -B(\epsilon/k)\,\bar{C}_f \bar{C}_o^{0.5} \tag{2}$$

Again, using the centerline experimental data for $\bar{U}_j/\bar{U}_e = 10$, the value $B = 4$ gave the best agreement. These results also are shown in Fig. 1 and differ only slightly from those obtained using Eq. (1). Figure 1 also shows computed distributions of

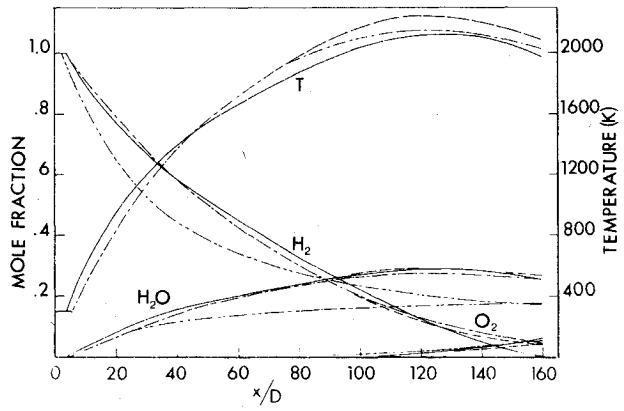

Fig. 1 Centerline values of temperature and mole fractions for $\bar{U}_j/\bar{U}_e = 10$. —— experiment,[10] –·– calculation using (1), - - - calculation using (2), – – – calculation with no reaction.

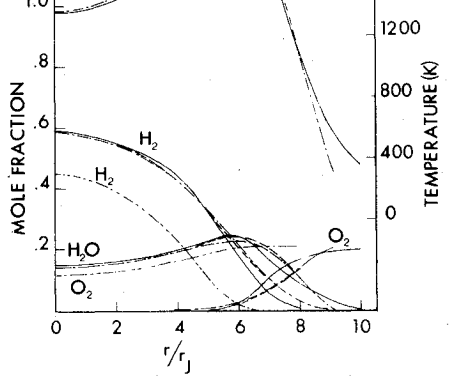

Fig. 2 Radial variation of temperature and mole fractions at $x/D = 40$ for $\bar{U}_j/\bar{U}_e = 10$. —— experiment,[10] legend as in Fig. 1.

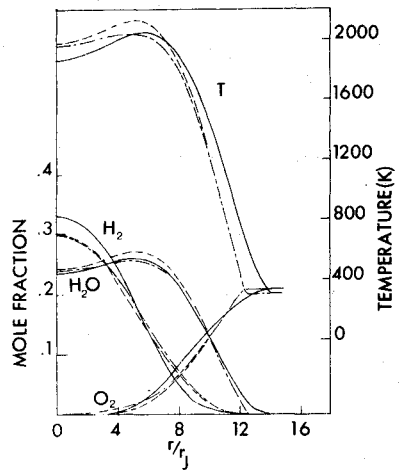

Fig. 3 Radial profiles at $x/D=80$ for $\bar{U}_j/\bar{U}_e=10$, legend as in Fig. 1

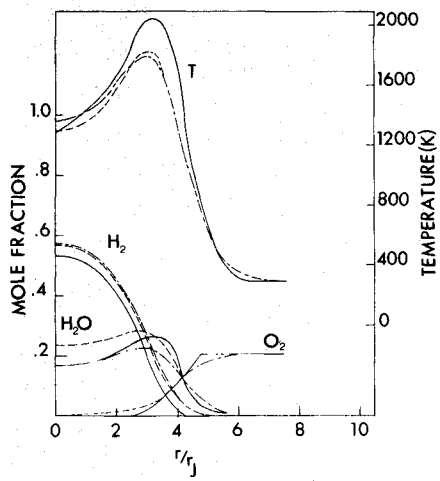

Fig. 5 Radial profiles at $x/D=40$ for $\bar{U}_j/\bar{U}_e=2$, legend as in Fig. 1.

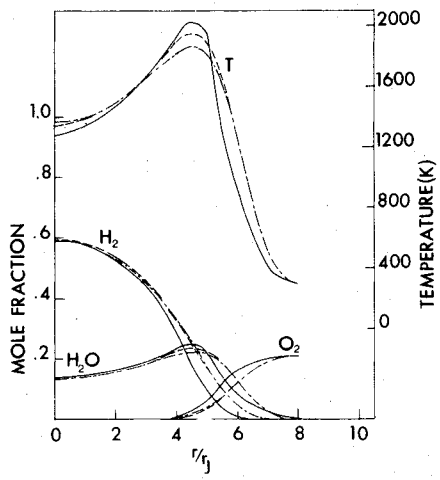

Fig. 4 Radial profiles at $x/D=40$ for $\bar{U}_j/\bar{U}_e=5$, legend as in Fig. 1.

the mole fraction of H_2 and of O_2 for the corresponding nonreacting case.

Radial profiles of temperature and mole fraction at $x/D=40$ and 80 are shown in Figs. 2 and 3, respectively. Here x is the axial distance from the hydrogen jet orifice and D is the jet diameter. The computed profiles using Eqs. (1) and (2) differ only slightly, and this occurs in the flame, as would be expected. Calculations also were made for $\dot{w}_f=-A_I(\epsilon/k)\bar{C}_f^2\bar{C}_o$. These indicated that the location of maximum temperature was shifted substantially toward the centerline, however.

Figure 2 also shows computed mole fraction profiles of H_2 and O_2 for mixing only, with no reaction. These profile shapes are similar to those for the reacting case, but the width of the mixing region is smaller for the nonreacting flow. The calculated mass fraction of hydrogen atoms on the centerline is 0.130 for the reacting case, and is 0.0539 for the nonreacting case. Similar results previously have been obtained experimentally.[13,14]

Computations next were made for the data of Kent and Bilger at $\bar{U}_j/\bar{U}_e$ equal to 5 and 2. The values of A and B obtained from the curve fits of the data for $\bar{U}_j/\bar{U}_e=10$ again were used. Radial profiles for $\bar{U}_j/\bar{U}_e=5$ and $x/D=40$ are shown in Fig. 4. The calculations evidently are consistent with the experimental data, and again Eqs. (1) and (2) produce nearly the same results. Finally, Fig. 5 shows the corresponding results for $\bar{U}_j/\bar{U}_e=2$ and $x/D=40$. Axial distributions for these two cases, like those shown in Fig. 1, indicate about the same degree of agreement with experiment as for $\bar{U}_j/\bar{U}_e=10$.

It is interesting that the simple estimate of Eq. (1) was able to produce results that are consistent with the experimental data examined here. This suggests that estimates of this form may provide useful results for other reacting flows. Such a form would seem to be restricted to turbulent flows that can be characterized by a single velocity scale and a single length scale at each point, as implied in the formulation of Eq. (1). It also must be anticipated that A will not be the same for all cases. More comparison with data for different types of flow obviously is required to clarify these points.

After this work had been completed, the author's attention was called to the work of Bush et al.[15] (The present study first appeared as a part of Ref. 16.) In their analysis of a diffusion flame in a two-dimensional turbulent shear flow, a rate equation was used that is closely similar to Eq. (1), the only difference being that the mean principal strain rate was used instead of ϵ/k. These are almost the same thing, however, as the characteristic times of the turbulence macroscale and of the mean flow should be of the same order in a turbulent flow driven by shear, if no other characteristic lengths or velocities are present. Thus, the present study may be considered as an application of the same theme, in which that theme is related to particular experimental conditions.

Acknowledgment

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Second-Order Unsteady Stagnation-Point Boundary-Layer Solutions

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I. Introduction

THE present work extends the second-order boundary-layer theory of Van Dyke[1] to unsteady flows. The work at the second-order level in unsteady flows is needed to gain insight into the effects of lower Reynolds number, encountered in diverse engineering applications. To illustrate the structure of second-order unsteady flow equations, several specific examples can be considered depending upon the type of unsteadiness in the basic potential flow for a given geometry of the body. For simplicity, one can study the situations where the solutions for the first-order (Prandtl) boundary-layer equations are known. In particular, the stagnation-point flow, owing to its importance, has been studied by several workers. The basic potential flow which admits self-similarity has been studied by Yang.[2,3] The periodically oscillating basic potential flow has been studied by Lighthill[4] and Ishigaki.[5] In the present work, we study the second-order boundary-layer equations near plane and axisymmetric stagnation points when the basic potential flow admits self-similarity. The case of a periodically oscillating stream will be reported in a later communication.

II. Second-Order Equations

Consider an unsteady laminar incompressible flow of a viscous fluid past a two-dimensional or axisymmetric body. The oncoming stream is assumed irrotational. Let the nondimensional streamwise and normal coordinates be s and n, respectively, and t is the time. The nondimensional velocity components in s and n directions are u and v. The static pressure is p and R is the characteristic Reynolds number.

The governing equations for second-order unsteady boundary-layer theory are obtained[6] from the Navier-Stokes equations by employing the method of matched asymptotic expansions with a perturbation parameter $R^{-1/2}$ as R approaches infinity. It is found that the unsteady flowfield can be described by the same two length scales (outer and inner) as in steady flows.[1] The outer expansions for u, v and p are of the type

$$u = U_1(s,n;t) + R^{-4}U_2(s,n;t) + \ldots \tag{1}$$

The inner expansions for u, $R^{1/2}v$, and p in terms of inner (Prandtl) variable $N = nR^{1/2}$ are of the type

$$u = u_1(s,N;t) + R^{-1/2}u_2(s;N;t) + \ldots \tag{2}$$

Substituting the inner and outer expansions (1) and (2) in the Navier-Stokes equations we get the equation for successive approximations. The matching of inner and outer solutions in the overlap region leads to the following first and the second-order boundary-layer equations[6]

First-order boundary-layer equations:

$$(r^j u_1)_s + (r^j v_1)_N = 0 \tag{3}$$

$$u_{1_t} + u_1 u_{1_s} + v_1 u_{1_N} - u_{1_{NN}} = U_{1_t} + U_1 U_{1_s} \tag{4}$$

$$u_1 = v_1 = 0 \qquad \text{at} \qquad N = 0 \tag{5a,b}$$

$$u_1 \sim U_1 \qquad \text{as} \qquad N \to \infty \tag{5c}$$

Second-order boundary-layer equations:

$$[r^j(u_2 + u_1 Nj\cos\theta/r)]_s + r^j[v_2 + (K + j\cos\theta/r)Nv_1]_N = 0 \tag{6}$$

$$u_{2_t} + (u_1 u_2)_s + v_1 u_{2N} + v_2 u_{1N} - u_{2NN}$$
$$= K[-Nu_{1_t} + (Nu_{1N})_N - v(Nu_1)_N] - [KNU_1^2$$
$$+ K\int_N^\infty U_1^2 - u_1^2 dN]_s + u_{1N} j\cos\theta/r + U_{2t} + (U_1 U_2)_s, \tag{7}$$

$$u_2 = v_2 = 0 \qquad \text{at} \qquad N = 0 \tag{8a,b}$$

$$u_2 \sim U_2 - KNU_1 \qquad \text{as} \qquad N \to \infty \tag{8c}$$

In these equations U_1 is used to denote $U_1(s,0,t)$ the basic potential flow and U_2 to $U_2(s,0,t)$ the perturbation to basic potential flow called the displacement speed. K is the longitudinal surface curvature of the body, θ the angle between axis of symmetry and the tangent to meridian curve at any point, r the radius of body and j is a number equal to zero for two dimensional flow and unity for axisymmetric flow. In the previous equations, terms proportional to K arise due to longitudinal curvature, $j\cos\theta/r$ due to transverse curvature and U_2 due to displacement effect.

III. Analysis Near Stagnation Regions

Yang[2,3] has shown that the first-order boundary-layer equations admit similarity when basic potential flow velocity at the wall U_1 is of the form

$$u_1(s,0;t) = s/(1-\alpha t) \tag{9}$$

where α is a constant. He has further shown that for a two-dimensional stagnation region solutions exist only for $\alpha \geq -3.175$ as for $\alpha = -3.175$ the first order skin friction vanishes.

In the neighborhood of plane and axisymmetric stagnation points K, r and $\cos\theta$ may be represented by $K \sim 1$, $r \sim s$ and

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